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WHAT MAKES A CRYSTAL "STIFF" ENOUGH FOR THE MOESSBAUER EFFECT?

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ABSTRACT: We show that the difficulties of interpreting the Moessbauer effect as a coherent lattice (phononic) phenomenon can be surmounted by relating it to a "superradiant" behavior of the plasma of nuclei of a crystal. As a result a "generalized" Debye-Waller factor is seen to emerge for determining the intensity of the effect.

Since its discovery in 1957 [1] the Moessbauer effect has not ceased to capture the fancy of all those who have thought about it in any depth. As well known the presently accepted theoretical understanding [2] goes back to Lamb [3] and Dicke [4], and was worked out in great and effective detail in the few years following its discovery.

In a nutshell, the explanation of the surprising behavior of a particular crystal that recoils collectively against the electromagnetic decay of a single nucleus lies in the fact that the nucleus is, so to say, an actor in a collective drama that manifests itself in the phononic excitations of the crystal, so that in a well built lattice each actor loses its identity to become part of the whole representation. Provided this collective behavior is not perturbed, i.e. none of the actors gets too excited and comes out to the front stage, quantum mechanics assures us that the process is coherent and its probability can be calculated from the Debye-Waller factor W as [2]

$$e^{-2W}=|\langle i|e^{i\vec{k}\vec{x}_N}|i\rangle|^2, \tag{1}$$

where $|i\rangle$ is the initial phononic state of the lattice, $\vec{k}$ is the momentum of the emitted photon, and $\vec{x}_N$ is the coordinate of the emitting nucleus, that can be linearly expressed in terms of the normal modes of the lattice vibrations. It has been shown that, under rather general conditions, one can express W as [5, 6] (we shall use throughout the natural units $\hbar=c=1$)

$$W=\frac{K^2}{2m_N}\frac{1}{K_B\theta_D}\frac{3}{4}\left[1+4\left(\frac{T}{\theta_D}\right)^2\int_0^{\theta_D/T}\frac{u\,du}{e^u-1}\right] \tag{2}$$

where θ_D is the Debye temperature. It should be pointed out that, even in such simple form, the theory has proven to be extremely successful.

As for the interpretation of (1), it is quite straightforward: it simply represents the probability that the electromagnetic wave of momentum $\vec{k}$ emerging from the decaying nucleus does not excite the initial phononic state. This is clearly the necessary condition for the lattice to recoil collectively against the photon, thus producing a photon energy distribution of great purity, unaffected by the Doppler shift that is inevitable in the decay in vacuum. The cooperative phenomena of the lattice have conspired to change drastically the arena in which the radioactive nucleus emits its photon: the crystal being very different from free space, i.e. the vacuum.

All this sounds very reasonable, but is it really? Let's make some estimates. The resolution power of our photon, as is well known, is of the order of its wave length; and taking a typical γ-transition of energy ω of a few tens of KeV, say 20 KeV, one has $\lambda=2\pi/\omega \simeq 5 \ 10^{-9}$ cm, an order of magnitude smaller than the typical lattice constant. Please note that the Moessbauer effect has been also shown to hold for transitions of the order of one hundred KeV.

In terms of momentum, the γ energy ω must be compared with the maximum momentum of the first Brillouin region ($\pi/a \simeq 1$KeV, for a $\simeq 4\mathring{A}$) which is also the maximum momentum of the lattice vibrations (the phonons). Thus we are led to the conclusion that the photon explores much more of the emitting nucleus's space distribution than can be accounted for by the phonons; and if this is the case, as it is difficult to deny, we must also conclude that the Debye-Waller factor (1) cannot be the whole story, for there is much more, not only the phonons, that ought to be kept nice and quiet: the quantum fluctuations that are necessary to describe the behavior of the nuclei at scales small compared with the lattice spacing a.

But a little reflection convinces us that, if phonons are all there is in a lattice and the lattice interactions have an ultraviolet cut-off at the first Brillouin zone, for momenta bigger than π/a the dynamics of the nucleus must be <<asymptotically free>> [7], i.e. it must be the same as the dynamics in vacuum, thus losing the Moessbauer effect altogether! Incidentally, this type of reasoning is also involved in the outright rejection, that is currently made by a large part of the scientific community, of the phenomena of "cold" nuclear fusion, whose implication is that fusion in vacuum must be drastically different from fusion in the Pd-lattice [8].

Also time considerations appear to lead us into trouble, for the typical time of the coherent recoil process is given by $\Delta t = E_R^{-1} \sim 10^{-13}$ sec ($E_R = \omega^2/2m_N \simeq 4\ 10^{-3}$eV is the classical recoil energy), during this time the phononic information that travels at the speed of sound ($v \simeq 10^6$cm sec^{-1}) will possibly correlate the nuclei in domains of the size of about 10 Å, which is clearly much too small: we seem to need the speed of light rather than the speed of sound.

However, as the Moessbauer effect is as incontrovertible as anything is in physics, we must try to understand where in the dynamics of QED arises the possibility of making the lattice "stiff" enough to support this fascinating fact of nature. It is the purpose of this paper to show that this possibility concretely exists in the recently developed Quantum Field Theory of "Superradiance" [9, 10], and that the Moessbauer effect represents, perhaps, the most relevant and natural confirmation so far of its basic ideas.

In this theoretical approach the nuclei constitute a "plasma" that is collectively represented by a quantum wave-field $\Psi(\vec{x}, \vec{\xi}; t)$, where $\vec{x}$ is the variable describing their equilibrium positions (the lattice) and $\vec{\xi}$ denotes the displacements from the above positions, (naturally, one has $\vec{x}_N = \vec{x} + \vec{\xi}$). For small displacements $\vec{\xi}$ the vibrations of the nuclei are controlled by the "plasma-frequency", which in the simplest plasma model is

$$\omega_p = \frac{Q}{(m)^{1/2}} \left(\frac{N}{V}\right)^{1/2} , \qquad (3)$$

where Q is the charge, m is the mass and (N/V) is the density of the nuclei. In ref. [10] it has been shown that for the simplest plasma the following results hold:

(i) the wave-field can be written as

$$\Psi(\vec{x}, \vec{\xi}; t) = \Psi_o(\vec{x}, \vec{\xi}; t) + \eta(\vec{x}, \vec{\xi}, t) \qquad (4)$$

where $\Psi_o(\vec{x}, \vec{\xi}; t)$ is a complex c-number wave-function such that

$$|\Psi_o(\vec{x}, \vec{\xi}, t)|^2 \simeq 0(N/V), \qquad (5)$$

while $\eta(\vec{x}, \vec{\xi}, t)$, the field of quantum fluctuations, is in general $0\,[\,|\Psi_o(\vec{x}, \vec{\xi}, t)|/\sqrt{N}]$;

(ii) the wave-function $\Psi_o(\vec{x}, \vec{\xi}; t)$ is $\vec{x}$-independent within space domains of at least the size of the e.m. field wave-length associated with the plasma frequency ω_p,

$$\lambda_p = \frac{2\pi}{\omega_p} , \qquad (6)$$

such space regions are called "coherence domains";

(iii) at temperature T=0, within a coherence domain, all charges oscillate in phase performing oscillations of well defined amplitude, depending on the anharmonicities of the real system. Such coherent charge oscillations are also in phase with a peculiar mode of the e.m. field, of wave length λ_p and frequency $\omega < \omega_p$, that remains trapped in matter;

(iv) the coherent e.m. field interacts also with the quantum fluctuations $\eta(\vec{x}, \vec{\xi}; t)$, producing gaps in the energy spectrum. When T increases the quantum fluctuations get excited with a Boltzmann spectrum up to the point when the condensed phase described by Ψ_o

is totally depleted, thus leading to a phase-transition.

It should now be clear that if the plasma of nuclei, of typical plasma frequency $\omega_p \sim 1\text{eV}$ and $\lambda_p \simeq 1\mu$, is in a superradiant state, the small scale vibrations in a fully coherent state will be described by a collective quantum oscillator wave-function of (complex) amplitude α. Thus the coherent recoil probability will be given by a "generalized" Debye-Waller factor

$$|\langle i\alpha|\, e^{i\vec{k}(\vec{x}+\vec{\xi})}\, |i\alpha\rangle|^2 = e^{-2W} \exp - \frac{\vec{k}^2|\alpha|^2}{3m_N\omega_p} \quad , \tag{7}$$

where the usual Debye-Waller factor is multiplied by an analogous term due to the small scale coherent oscillations of the plasma of nuclei. Note that by defining the "plasma-temperature"

$$K_B\Theta_p = \frac{\omega_p}{|\alpha|^2}\left(\frac{9}{4}\right) \quad , \tag{8}$$

one gets $\Theta_p \simeq 2.5\ 10^4/|\alpha|^2$ °K, a very high temperature unless $|\alpha|^2$ is also a rather large number. We may try to estimate the size of α as that of the amplitude of plasma oscillations for which the harmonic approximation breaks down and the superradiant process gets out of tune [10]. One has $|\alpha|^2/m_N\omega_p = r_o^2$, where r_o is such amplitude (a small fraction of the Bohr radius $a_o \simeq .53\text{Å}$). For instance for $r_o \simeq .1\text{Å}$, $m_N = 50$ GeV and $\omega_p = 1\text{eV}$, one has $|\alpha|^2 \simeq 10^2$, getting a reasonable plasma temperature of the order of 250 °K. This means that the new factor appearing in (7) is relevant as well for the determination of the intensity of the Moessbauer effect.

As a last exercise we would like to compare the minimum number N_c of coherent recoiling nuclei, i.e. those contained in a coherent domain, with known experimental limits. On one hand we have

$$N_c = \left(\frac{N}{V}\right) \lambda_p^3 \simeq 10^{10} \quad , \tag{9}$$

obtained by setting $(N/V) \simeq 10^{22}$ and $\lambda_p \simeq 1\mu$. On the other hand taking, for instance, the experiment of Pound and Rebka [11], where a sensitivity of about $2\ 10^{-16}$ has been achieved, the experimental results imply that the Doppler shift δ (N_r is the number of recoiling nuclei) must satisfy

$$\delta = \frac{1}{N_r} \left(\frac{\omega}{m_N}\right) < 2\ 10^{-16} \quad ; \tag{10}$$

and setting ω=15 KeV, m_N=50 GeV, we obtain

$$N_r > 1.5\ 10^8 \quad , \tag{11}$$

compatible with (8) [12].

We conclude by stressing that the mysterious nature of the Moessbauer effect, that engenders a strong violation of <<asymptotic freedom>> in a crystal, has been resolved by assuming that the plasma of nuclei undergoes a "superradiant" dynamical evolution. We believe that this is a further piece of the jig-saw puzzle of coherent electromagnetism in condensed matter that goes into place.

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